MATH 105A and 110A Review: Elementary matrices and row operations

Facts to Know:

Elementary row operations on a matrix A:

1. **(Row switching)**

2. **(Row addition)**

3. **(Row multiplication)**

The matrix A is in **echelon form** if the following three properties hold:

1. Rows of zeros are below any
2. Any leading entry of a row is in a column to the right of the leading entry of the
3. All entries in a column below a leading entry are

Picture of echelon form:

$$\begin{bmatrix} \blacksquare & * & * & * \\ 0 & \blacksquare & * & * \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & \blacksquare & * & * & * & * & * & * & * & * \\ 0 & 0 & 0 & \blacksquare & * & * & * & * & * & * \\ 0 & 0 & 0 & 0 & 0 & \blacksquare & * & * & * & * \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \blacksquare & * \end{bmatrix}$$

- The $\blacksquare$'s correspond to non-zero numbers and are called
- The columns with $\blacksquare$'s are called

If the matrix A is in echelon form and additionally satisfies the following two conditions, then A is in **reduced echelon form**:

4. The leading entry in each nonzero row is
5. Each leading 1 is the only nonzero entry in its

Examples:

1. Identify which matrices are in echelon form, reduced echelon form, or neither.

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

2. Reduce A to reduced echelon form.

$$A = \begin{bmatrix} 1 & 3 & 1 & 3 & 3 \\ 0 & 2 & 0 & 2 & 2 \\ 0 & 0 & 0 & 3 & 3 \end{bmatrix}.$$

Facts to Know:

An **elementary matrix** E is obtained by applying one of the to the matrix I.

Each elementary row operation can be carried out by multiplying A from by the corresponding

Examples:

3. Carry out row operation $R_1 + 2R_2 \to R_1$ on matrix A using an elementary matrix, where

$$A = \begin{bmatrix} 1 & 1 \\ 1 & -1/2 \end{bmatrix}.$$